LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – NOVEMBER 2007

MT 5400 - GRAPH THEORY

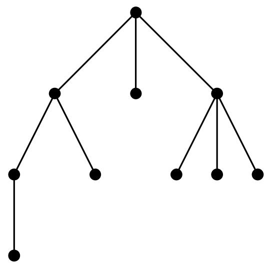
AB 9

Date : 03/11/2007 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART A

Answer all the questions. Each question carries 2 marks.

01. Give an example of a complete graph which is 4-regular also.
02. The only regular graph of degree 1 is K_2 -- True or false? Justify your answer.
03. Give an example of a graph with a vertex of maximum degree 56.
04. What is the relation between the cardinalities of vertex sets of two isomorphic graphs?
05. Give an example of a closed walk of even length which does not contain a cycle.
06. Draw all non-isomorphic trees on 6 vertices.
07. Give an example of a graph which has a cut vertex but does not have a cut edge.
08. Prove that in a tree every edge is a cut edge.
09. Let $G = (V, E)$ be a (p, q) graph. Let $v \in V$ and $e \in E$. Find the number of vertices and edges in $G - v$ and $G - e$.
10. Determine the centre of the following tree.

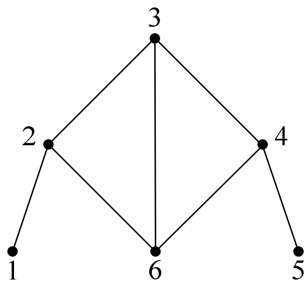


PART B

Answer any 5 questions. Each question carries 8 marks.

11. (a). Prove that in any graph the number of vertices of odd degree is even.
    (b). Draw the eleven non-isomorphic sub graphs of the complete graph on 4 vertices.

    (4+4)
12. Prove that the maximum number of edges among all graphs with p vertices, where p is odd, with no triangles is $[p^2 / 4]$, where $[x]$ denotes the greatest integer not exceeding the real number x.
13. (a). Obtain the complement of the following graph.



(b). Let G be a k-regular bipartite graph with bipartition (X, Y) and $k > 0$. Prove that $|X| = |Y|$.

(4 + 4)

14. (a). Prove that a graph G is connected if and only if for any partition of V into subsets V_1 and V_2 there is an edge with one end in V_1 and the other end in V_2.
(b). Show that if G is disconnected then G^C is connected.

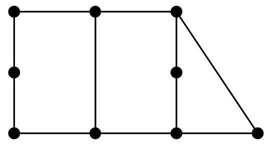
(4 + 4)

15. (a). Prove that any self - complementary graph has $4n$ or $4n+1$ vertices.
(b). Prove that a graph with p vertices and $\delta \geq \frac{p-1}{2}$ is connected.

(4 + 4)

16. Prove that a graph G with at least two points is bipartite if and only if all its cycles are of even length.

17. (a). Determine the chromatic number of the following graph.



(b). Prove that every tree has a centre consisting of either one vertex or two adjacent vertices. (4 + 4)

18. Let G be a block. Then prove that any two vertices of G lie on a common cycle.

PART C

Answer any 2 questions. Each question carries 20 marks.

19. Let G_1 be a (p_1, q_1)-graph and G_2 a (p_2, q_2)-graph. Show that
    1. $G_1 \times G_2$ is a $(p_1 p_2, q_1p_2 + q_2p_1)$-graph and
    2. $G_1[G_2]$ is a $(p_1 p_2, q_1p_2^2 + q_2p_1)$-graph.

20. (a). Let G be graph with $p \geq 3$ and $\delta \geq \frac{p}{2}$. Then prove that G is Hamiltonian.
(b). Let v be a cut-vertex of a connected graph. Then prove that the there exists a partition of $V - \{v\}$ into subsets U and W such that for each $u \in U$ and $w \in W$, the point v is on every (u, w) - path.

(15+5)

21. Let G be a (p, q)-graph. Prove that the following statements are equivalent.
    1. G is a tree.
    2. Any two vertices of G are joined by a unique path.
    3. G is connected and $p = q + 1$.
    4. G is acyclic and $p = q + 1$.

22. (a). Obtain Euler's formula relating the number of vertices, edges and faces of a plane graph.

(b). Prove that every planar graph is 5-colourable. (10+10)

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